Intermodulation products

Dynamic Force Quadratures

Intermodulation Products offers a new way to study tip-surface interactions in dynamic Atomic Force Microscopy (AFM). The amplitude dependence of conservative and dissipative forces felt by an oscillating cantilever reveals both the elastic and viscous nature of the material surface.

Quasi-static force curves

The tip-surface force in AFM is usually considered to be in quasi-static equilibrium with the cantilever force, and proportional to cantilever deflection $F_{TS}=F_{cant}=k\,d$. A measurement of cantilever deflection as the probe is moved toward and away from the surface, is transposed (under many assumptions) to the tip-surface force as a function of the tip position $F(z)$. This 'force curve' is fitted to models from contact mechanics to extract the elastic modulus of the surface.

The quasi-static approach to force measurement breaks down as the cantilever begins to move faster, where inertial and viscous forces must also be included in the full description of the cantilever dynamics. Furthermore, the quasi-static method can not reveal anything about the viscous response of the surface, as viscous forces are proportional to velocity.

Dynamic force measurement

Viscousity can be revealed through a dynamic measurement of force, where the cantilever deflection is analyzed as a function of time $d(t)$. Actually, it is much more advantageous to measure the cantilever deflection as a function of frequency. Multifrequency lock-in measurement greatly improves the signal-to-noise ratio. We measure the quadrature response $\hat{d}(\omega)=d_I(\omega)+i\,d_Q(\omega)$ or the Fourier cosine and sine components of the cantilever deflection at each frequency ω . This response is easily convert to $d(t)$, as the time and frequency domains are related to one-another via the Fourier transform and its inverse.

Analysis of the tip-surface force is much easier in the frequency domain, where force and deflection have a simple linear relation [1],

$$\hat{d}(\omega)=\hat{\chi}(\omega)(\hat{F}_{TS}+\hat{F}_{drive}) \qquad (1)$$

Here $\hat{\chi}(\omega)$ is the cantilever's linear response function, which is determined by a non-invasive calibration procedure. Solving Eq. (1) for the Fourier coefficients of the tip-surface force we find,

$$\hat{F}_{TS}=\hat{\chi}^{-1}(\omega)(\hat{d}(\omega)-\hat{d}_{free}(\omega)) \qquad (2)$$

where the drive force $\hat{F}_{drive}=\hat{\chi}^{-1}\hat{d}_{free}(\omega)$ is given by the measured free motion, far from a surface where $F_{TS}=0$.

Measurement in the frequency domain is particularly advantageous close to a high quality-factor resonance. On resonance the response function gives a factor Q larger deflection, than that due to a static force $|\hat{\chi}(\omega_0)/\chi(0)|=Q$. Because dynamic force measurement is so sensitive near resonance, one is essentially limited to measuring frequency components of the force in a narrow frequency band near resonance.

Dynamic force curves

Intermodulation Products has developed a method to analyze and visualize tip-surface force in this narrow frequency band near resonance. From the multifrequency measurement of the tip-surface force near resonance, we obtain two dynamic force curves showing the Fourier coefficients of the tip-surface force at ω_0 , as a function of oscillation amplitude. These Fourier coefficients are transformed to a rotated frame, such that one component is in-phase with the motion of the cantilever, and the other quadrature to the motion, or in phase with the velocity.

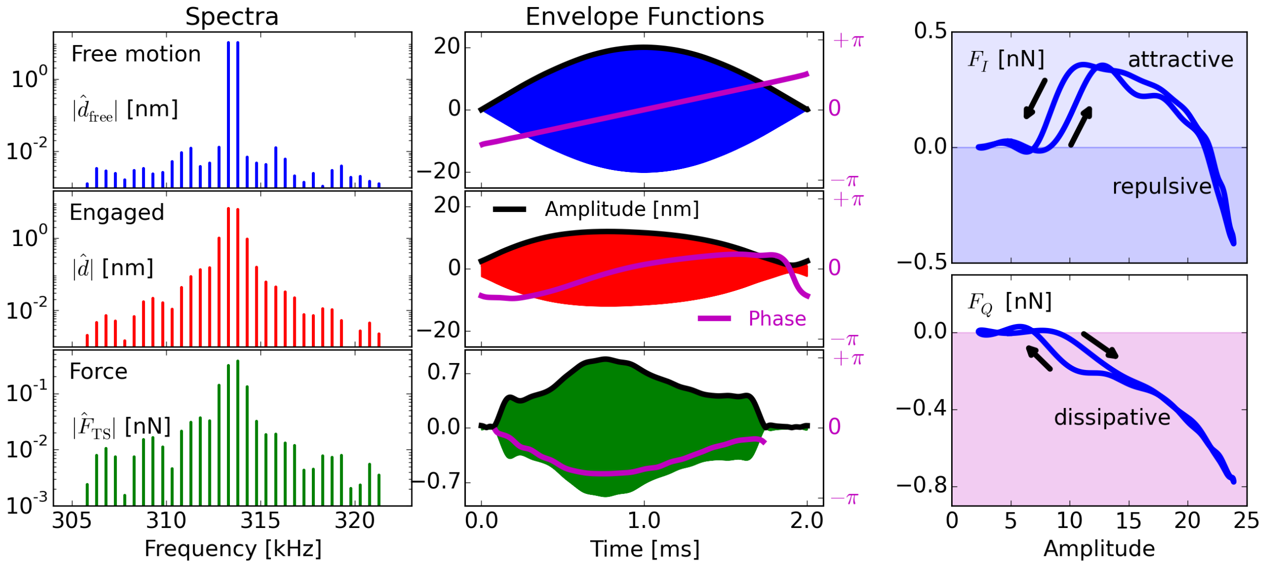

Figure 1: The spectrum of free motion and engaged motion are combined with the calibrated transfer function as in Eq.(2) to get the spectrum of the tip surface force. From the motion spectrum and force spectrum we extract the slowly varying envelopes, or time-domain modulation functions, by simple down-shifting and inverse Fourier transform. Thus, for each fast oscillation of the cantilever, we can follow the amplitude and phase of both motion and force. Knowing the phase of both envelope functions, we can plot the force which is in phase with the motion $F_I(A)$ *, and its quadrature force* $F_Q(A)$ *, as functions of the amplitude of oscillation* A *.*

$$d(t)=A\cos(\omega_0 t)$$
$$F_I(A)=\frac{1}{T}\int_0^T F_{TS}(d,\dot{d})\cos(\omega_0 t)$$
$$F_Q(A)=\frac{1}{T}\int_0^T F_{TS}(d,\dot{d})\sin(\omega_0 t) \quad (3)$$

in the frequency domain we write,

$$\hat{d}=A+i0$$
$$\hat{F}_{TS}(A)=F_I(A)+iF_Q(A) \quad (4)$$

Intermodulation near a high-Q resonance offers a rapid way to measure the amplitude dependence of the force quadratures[2]. The measured spectra can be directly transformed into the two force quadrature curves as demonstrated in fig. 1. We can think of $F_I(A)$ and $F_Q(A)$ as the 'force curves' of dynamic AFM.

The dynamic force curves tells us a great deal about the tip-surface interaction. Unlike the quasi-static force curve $F(z)$ they do no tell us the force at a specific location z, but rather the integrated force on a single oscillation cycle with amplitude A. The in-phase force F_I is conservative, meaning that it describes an elastic response of the surface. When F_I is positive the elastic part of the tip-surface force is dominantly attractive during the single oscillation cycle. Similarly, negative F_I means that the elastic force is dominantly repulsive. The dissipative quadrature F_Q tells us about the viscous response of the surface. In fig. 1 we see that F_Q has significant magnitude, even when F_I is positive. The attractive forces pull the surface upward, and due surface viscosity, energy in the cantilever is dissipated by the oscillating surface.

The force quadratures are measured for both increasing and decreasing amplitude during the full modulation cycle. With soft materials one often sees hysteresis in the dynamic force curves as shown in fig. 1. The surface is lifted by attrictive forces and when the contact is broken, the surface does not fully relax before the next oscillation lifts the sufrace again. Repeated taps cause a lifted time-average surface position.

References

1) The role of nonlinear dynamics in quantitative atomic force microscopy. D. Platz, D. Forchheimer, E. A. Tholén and D. B. Haviland. Nanotechnology **23**, 265705 (2012).

2) Interpreting motion and force for narrow-band intermodulation atomic force microscopy. D. Platz, D. Forchheimer, E. A. Tholén and D. B. Haviland. Beilstein J. Nanotechnol. **4**, 45 (2013).